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A NONLINEAR MOTOR-GEAR MODEL AND ITS APPLICATION TO LOAD-SHARING ANALYSIS OF WIND TURBINE YAWING MECHANISMS

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Keywords: Multi-motor-gear system, nonlinear dynamic modelling, load-sharing analysis, multi-body dynamics, wind turbine yawing mechanism.

Abstract. *Multi-motor-gear system can be used in special applications, where high torques are required. An inherent problem with such types of systems is the uneven load sharing among all paths, for which a dynamic model is desired for a better understanding of the problem. In this paper, a nonlinear dynamic model of the multi-motor-gear system is developed, in which non-linearity of gear meshing stiffness, gear geometric and assembly errors, and the motor characteristics are duly considered. The developed model was applied to the dynamic analysis of a wind turbine yawing mechanism, with focus on the system's dynamic behavior at low frequencies. Simulation results are included to show the effectiveness of the model in the analysis of load sharing. Moreover, preliminary parametric study reveals that the uneven load sharing becomes significant at low frequencies.*

1 Introduction

A multi-motor-gear system refers to a parallel shaft motor-gear arrangement in which an input gear meshes with multiple output gears, or, *visa versa*, multiple input gears mesh an output gear [1, 2]. Multi-motor-gear systems have some promising features such as high capability of torque transmission, low complexity of mechanism, and compactness. They can be found in applications requiring high torque in a compact structure, such as helicopters and wind turbines.

While multi-motor-gear systems can effectively transmit power, a fundamental problem is the uneven load sharing among all paths. As a matter of fact, the load applied on the system cannot be equally shared by all paths, due to influences of the non-linearity of the gear teeth, the manufacture and assembly error, etc. The uneven load sharing increases the noise level, reduces the efficiency and more seriously, shortens significantly the life expectancy. To make the load equally distributed among gears is thus a big issue in transmission design and development.

The multi-motor-gear system can be considered as the extension of multiple-parallel-transmissions, which have been studied with different approaches, based mainly on gear dynamics [3]. Krantz developed a static model to analyze the load sharing of split-path transmissions. A dynamic model was developed using rigid-body and lumped mass approximation by Dama, et al. [4]. The influence of manufacturing errors on the gear dynamics was studied in [5]. The carrier and gear manufacturing errors were considered in the modelling of planet gear trains [6]. An experimental study of the load distribution was reported in [7], showing that the mean value of loads are nearly equal, while the dynamic load varies largely. In these reported works, the nonlinear gear meshing was mostly modeled with a time-varying stiffness. Moreover, these studies were focused on the steady-state performance at high frequencies, the dynamic behavior at frequencies lower than the characteristic frequencies being seldom considered.

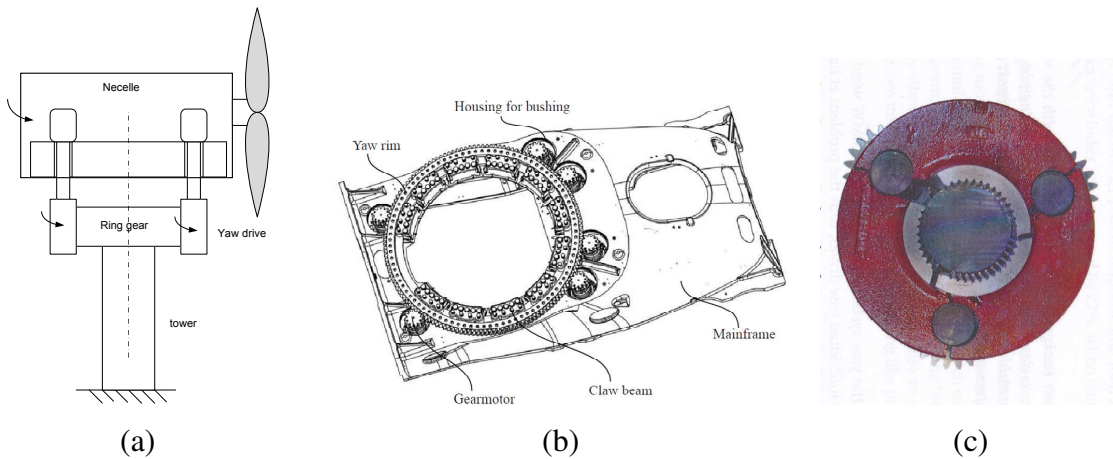


Figure 1: (a) Schematic drawing of a wind turbine, (b) bottom view of the yawing mechanism and (c) a broken gear in the yaw drive

In this paper, a dynamic model of the multi-motor-gear system is developed by considering the dynamics of gear, the stiffness of gear teeth and shafts as well as gear geometric errors such as backlash. The new model extends the existing nonlinear gear meshing model with new features by including the motors and payloads, so that the new model can better represent the dynamic performance of a real system. Simulations were conducted using the developed model for systems running at low frequencies.

The reported work was carried out with a special interest in the torsional vibrations of wind turbine gear mechanisms, particularly, the yawing mechanism. The yawing mechanism is a key system in a wind turbine, as illustrated in Fig. 1a, which ensures the turbine to yaw up against the wind to maximize the wind power. It works intermittently at low yawing speed. In large-scale wind turbines with capacity larger than 1MW, the yawing mechanism is driven with 4 to 8 yaw drives, as the one displayed in Fig. 1b. An example of a broken planetary carrier of a gear motor in a wind turbine is shown in Fig. 1c, for which the uneven load sharing is arguably thought as a major reason of failure. The system modelling will help better understand the load sharing and the influencing factors, and to improve the reliability of the wind turbine.

2 Dynamics Model

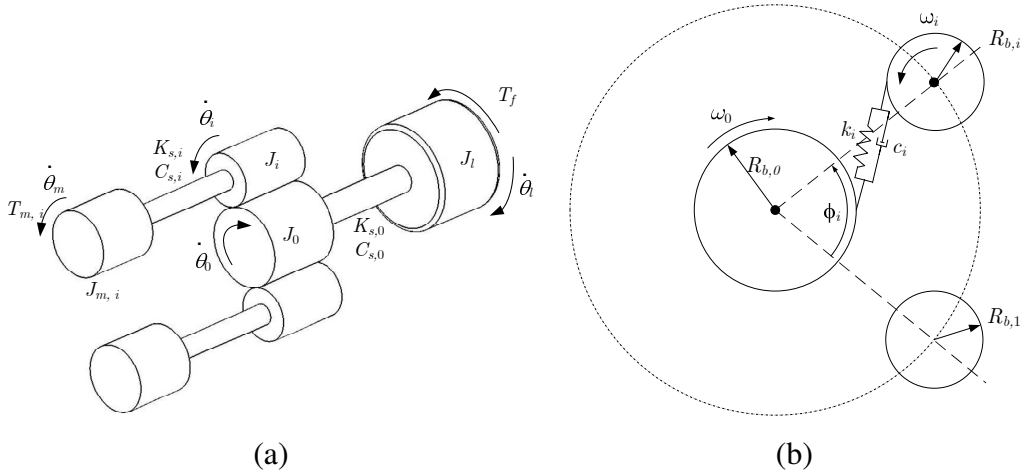


Figure 2: A multi-motor-gear system and meshing force model

A multi-motor-gear transmission system is schematically depicted in Fig. 2, where only two paths are shown in the figure for clarity. In the modelling, only torsional displacements of the systems are considered. All gears are assumed to be rigid, while the teeth are flexible. The gear meshing is characterized by springs of varying stiffness.

Let $\theta_{m,i}$ and θ_i be the rotation angles of the i th motor and planet gear, while θ_l and θ_0 are the rotation angles of the load and the sun gear. The equation of motion of the system can be written as

$$J_{m,i}\ddot{\theta}_{m,i} + C_{s,i}(\dot{\theta}_{m,i} - \dot{\theta}_i) + K_{s,i}(\theta_{m,i} - \theta_i) = T_{m,i}, \quad i = 1..N \quad (1a)$$

$$J_i\ddot{\theta}_i + C_{s,i}(\dot{\theta}_i - \dot{\theta}_{m,i}) + K_{s,i}(\theta_i - \theta_{m,i}) + F_{b,i}R_{b,i} = 0 \quad (1b)$$

$$J_0\ddot{\theta}_0 + C_{s0}(\dot{\theta}_0 - \dot{\theta}_l) + K_{s0}(\theta_0 - \theta_l) - \sum_{j=1}^N F_{b,j}R_{b0} = 0 \quad (1c)$$

$$J_l\ddot{\theta}_l + C_{s0}(\dot{\theta}_l - \dot{\theta}_0) + K_{s0}(\theta_l - \theta_0) = T_f \quad (1d)$$

where $F_{b,i}$ is the meshing force between the i th planet and the sun gear, while T_f is the force applied on the load side, including friction force. Moreover,

$J_{m,i}$ and J_l — moments of inertia of the i th motor and the load

$R_{b,i}$ and R_{b0} — radii of the base circles of the i th driving gear and the driven gear

$C_{s,i}$ and $K_{s,i}$ — damping coefficient and stiffness of the i th input shaft

C_{s0} and K_{s0} — damping coefficient and stiffness of the shaft connected to the load

F_i — gear meshing force along the line of action

If backlash and gear manufacturing errors are ignored, the meshing force can be expressed as

$$F_{b,i} = k_i u_i + c_i \dot{u}_i \quad (2)$$

with transmission error $u_i = R_{b,i}\theta_i - R_{b0}\theta_0$. With practical considerations, the transmission error is subject to manufacturing error e . In light of this, the transmission error is modified for general cases as

$$u_i = R_{b,i}\theta_i - R_{b0}\theta_0 + e \quad (3)$$

When backlashes are considered, the meshing force can be written as

$$F_{b,i} = k_i u_i + c_i \dot{u}_i + \tilde{F}_{b,i}(u_i, B) \quad (4)$$

where $\tilde{F}_{b,i}$ is the term considering the influence of backlash B .

The gear meshing stiffness k_i varies with the position of engagement. In the steady state, k_i can be regarded to change with time, that is

$$k_i = \bar{k}_i + \tilde{k}_i \quad (5)$$

where \bar{k} is the mean value of the stiffness, and \tilde{k}_i is a periodic function of stiffness whose mean value is null. With (5), equations (1a) to (1d) are transformed as

$$J_i J_{m,i} \ddot{\phi}_i + (J_{m,i} + J_i) C_{s,i} \dot{\phi}_i + J_{m,i} R_{b,i} c_i \dot{u}_i + (J_{m,i} + J_i) K_{s,i} \phi_i + J_{m,i} R_{b,i} \bar{k}_i u_i = f_{\phi,i} \quad (6)$$

$$J_0 J_i \ddot{u}_i + J_0 R_{b,i}^2 c_i \dot{u}_i + J_i R_{b0}^2 \sum c_j \dot{u}_j + C_{s,i} J_0 R_{b,i} \dot{\phi}_i - C_{s0} J_i R_{b0} \dot{\phi}_0 + J_0 R_{b,i}^2 \bar{k}_i u_i + J_i R_{b0}^2 \sum \bar{k}_j u_j + K_{s,i} J_0 R_{b,i} \phi_i - K_{s0} J_i R_{b0} \phi_0 = f_{u,i} \quad (7)$$

$$J_0 J_l \ddot{\phi}_0 + (J_l + J_0) C_{s0} \dot{\phi}_0 - J_l R_{b0} \sum C_j \dot{u}_j + (J_l + J_0) K_{s0} \phi_0 - J_l R_{b0} \sum \bar{k}_j u_j = f_{\phi 0} \quad (8)$$

with $\phi_0 = \theta_0 - \theta_l$. Moreover, $\phi_i = \theta_i - \theta_0$ for $i = 1..N$ and $\sum \equiv \sum_{j=1}^N$. The right-hand sides of equations are equivalent excitations, which are

$$f_{\phi,i} = -J_i T_{m,i} - J_{m,i} \tilde{k}_i R_{b,i} u_i; \quad (9)$$

$$f_{u,i} = -\tilde{k}_i(t) J_0 R_{b,i}^2 u_i - J_i R_{b0}^2 \sum \tilde{k}_j u_j + J_0 J_i \ddot{e}; \quad (10)$$

$$f_{\phi 0} = -J_0 T_f + J_l R_{b0} \sum \tilde{k}_j u_j; \quad (11)$$

The above system can be expressed in a matrix form

$$\mathbf{M}\ddot{\mathbf{q}} + \mathbf{C}\dot{\mathbf{q}} + \mathbf{K}\mathbf{q} = \mathbf{f} \quad (12)$$

The dimension of the matrix depends on the number of planet gears.

The natural frequency $\omega_n, n = 1, \dots, 2N + 1$ of the linear system on the left-hand side of equation (12) satisfies

$$\det(\omega_n^2 \mathbf{M} - \mathbf{K}) = 0 \quad (13)$$

from which the natural frequencies can be found.

In the formulation of the equation of the system, each gear is considered as a rigid body. The elastic nature of gear teeth is considered in the determination of gear meshing force. It is noted that there is another approach as reported in [8], where each tooth is treated as a separated body connected to the gear body by means of springs.

While different approaches are available to find the solution to the system, this work adopts the numerical approach for the solution of dynamic response and performance analysis.

3 Gear teeth stiffness in gear meshing

The gear meshing stiffness can be approximated by a Fourier series as

$$k_i(t) = \bar{k} + k_r \sum_{j=1}^L a_j \sin j(\omega_m t + \psi_{0,i}) \quad (14)$$

where ω_m is the meshing frequency and $\psi_{0,i}$ is the initial phase angle. It is noted that the meshing stiffness varies with the teeth profile.

4 Influence of backlash on gear meshing force

The force $F_{b,i}$ is in principle equal to the product of gear meshing stiffness and gear tooth deflection. However, it is also subjected to factors including gear backlash, manufacturing errors, etc.

Let B denote the backlash. Three cases can be classified according to conditions of gear meshing:

1. positive working condition, a situation in which the driving gear is in contact with the front side of the driven gear. Such a condition is expressed as $u_i(t) < 0$
2. negative working condition. In such a situation, a driven gear is in contact with the backside of the driving gear. This condition can be expressed as $u_i(t) > B$
3. non engagement, a situation of $0 < u_i(t) < B$.

Based on these three cases of gear meshing, the forces generated in gear meshing can be calculated by

$$F_{b,i} = \begin{cases} k_i(t)u_i(t) + c_i\dot{u}_i(t) & \text{if } u_i(t) < 0 \\ -k_i(t)u_i(t) + c_i\dot{u}_i(t) & \text{if } u_i(t) > B \\ c_i\dot{u}_i(t) & \text{if } 0 < u_i(t) < B \end{cases} \quad (15)$$

If the negative working condition can be ignored, a more convenient form will be presented as

$$F_{b,i} = k_i(t)(u_i(t) - B)(1 + \tanh(\lambda(u_i(t) - B)))/2 + c_i\dot{u}_i(t); \quad (16)$$

where λ is a large number, which takes the value of 10^8 in this work. The influence of backlash is found as

$$\tilde{F}_{b,i} = k_i(t)(u_i(t) - B)(\tanh(\lambda(u_i(t) - B)))/2 + k_i(t)(u_i(t) + B)/2; \quad (17)$$

Table 1: Gear Parameters

	planet gear	sun gear
Number of teeth	18	35
module[mm]	15	15
pressure angle[deg]	20	20
teeth width [mm]	270	270
MOI [kg.m ²]	1.09	15.7

5 Manufacturing Error Modelling

The dynamic behavior of the gear trains is under the influence of manufacturing errors, including the assembly error. The errors are classified into errors in the position, run-out and tooth-thickness. We adopt the concept of Total Composite Error (TCE) [9] of gear manufacturing in this study.

The error function of a gear is approximately sinusoidal for practical considerations. Pitch circle run-out will cause a sinusoidal error which is revealed as an output transmission error when meshed with a mating gear. Thus manufacturing error in terms of TCE can be modeled as

$$e = E_t \sin \omega_i t + E_p \sin n_i \omega_i t \quad (18)$$

where the amplitude of teeth profile error E_p and the TCE E_t , varying with gear precision and size, can be found by reference to gear manufacturing standards. Moreover, ω_i is the angular velocity of the input gear, which has n_i teeth.

The system of dynamic equations of the multi-motor-gear system is readily to solve, with the developed gear meshing model.

6 Simulations and Analysis

In what follows, we will use the developed model to simulate the dynamic performance, with interests in the load sharing at low frequencies. A few load sharing indices are defined for the dynamic analysis of the multi-path transmissions. Let $T_{q,i} = F_{b,i} R_{b,i}$ be the torque transmitted through the i th planet gear. From [10, 11], a dynamic load sharing factor is defined as

$$K_{ls,i} = \frac{T_{q,i}}{\sum T_{q,i}} \quad (19)$$

Upon the dynamic load sharing factors, a load-sharing index is further defined for the system as

$$R_{ls} = \frac{\max(K_{ls,i})}{\min(K_{ls,i})} \quad (20)$$

The minimum value of R_{ls} is bounded to 1. The smaller the index, the better the load distribution.

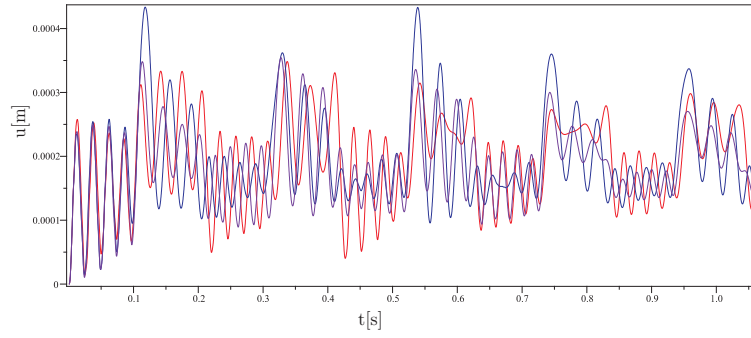
Of the two load factors, the dynamic load sharing factors show how individual gear transmits powers, while the latter indicates how large the unevenness of load distribution in a system.

6.1 Example 1. A three-motor-gear system

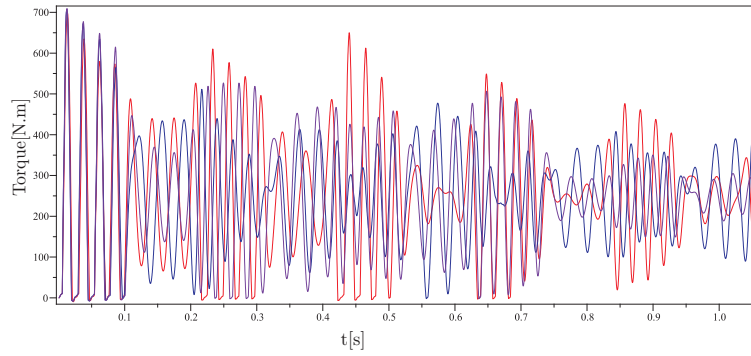
Two examples are included to demonstrate the new model. In the first example, the parameters of the system are given in Table 1, while simulation parameters are listed in Table 2. The first natural frequency is found as $\omega_n = 287 \text{ rad/s}$. A simulation was conducted with meshing frequency

Table 2: Simulation parameters

shaft damping coefficient [N.m/s]	$C_{s,i} = 1.98 \times 10^2$; $C_{s0} = 1.98 \times 10^3$
gear damping coefficient [N/s]	$c_i = 2.45 \times 10^3$
shaft stiffness[N.m/rad]	$K_{s,i} = 2.0 \times 10^6$; $K_{s0} = 2.0 \times 10^7$
gear meshing stiffness[N/m]	$\bar{k}_i = 3.5 \times 10^7$; $k_r = 2.0 \times 10^7$
payload [N.m]	$T_f = 750$



(a)



(b)

Figure 3: System simulation with $\omega = 30\text{rad/s}$, (a) transmission error, and (b) torques transmitted via different paths, indicated by different colors

$\omega = 30\text{rad/s}$. The transmission error and transmitted torques through each path are shown in Fig. 3. A phase plot of the system is shown in Fig.4, where the chaos phenomenon is displayed.

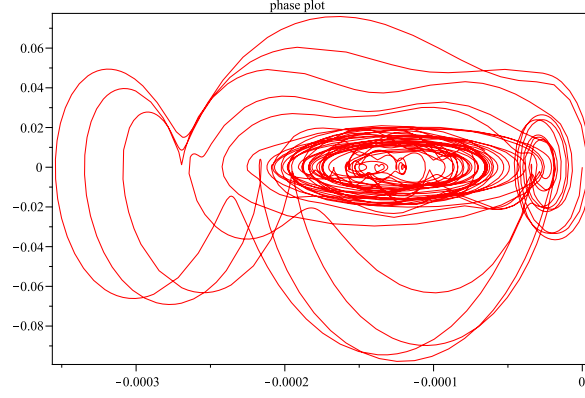


Figure 4: Phase plot of Example 1

At low frequency, the load sharing factors of three planets are 34.1%, 32.7% and 33.0%. The load sharing index of the system is equal to 1.04. The maximum torques applied on each planet are 650, 672, and 677 $N.m$, which account for 2.6, 2.69, and 2.70 times of the ideally distributed static load.

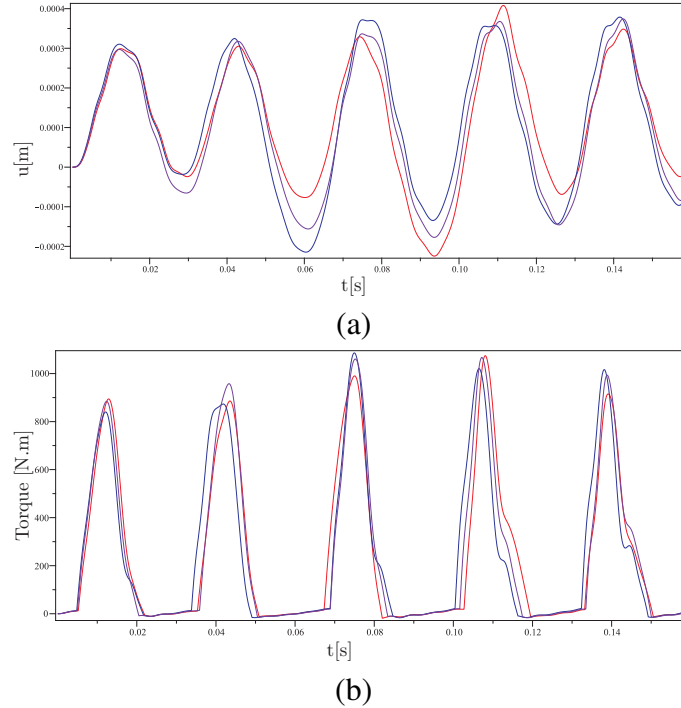


Figure 5: Simulation of $\omega = 200\text{rad/s}$, (a) transmission error, and (b) torques transmitted via different paths, indicated by different colors

In another simulation, the meshing frequency is increased to 200rad/s . The results are shown in Fig. 5. the load sharing factors of three planets are 32.8%, 33.5% and 33.5%. The load sharing

Table 3: Yawing mechanism parameters

	planet gear	ring gear
Number of teeth	11	143
module[mm]	16	16
pressure angle[deg]	20	20
MOI [kg.m ²]	0.13	546.2

index of the system is found as 1.02. Moreover, the higher the frequency, the better the load distribution.

6.2 Example 2. A wind turbine yawing mechanism

In this example, the developed method is demonstrated with a yawing mechanism of wind turbines. The system parameters are given in Table 3.

The motors have an equivalent moment of inertia equal to $10768 N.m^2$ and their output torques are $6000 N.m$. The shaft stiffness varies in the range of $1.40 \sim 1.54 \times 10^6 Nm/rad$. The damping ratios all set to 0.05 for simplicity. The gear meshing stiffness, based on FEA simulation, is found as $1.15 \sim 1.65 \times 10^5 Nm/deg$, or $8.8 \sim 11.8 \times 10^8 N/m$. As shown in figures 6a and b, the six yaw drives carry different loads, with the dynamic load-sharing factor ranging from 0.11 to 0.22, comparing an ideal sharing factor of $1/6$. The uneven load sharing shown in the simulation partially agrees with the measurement on a real system.

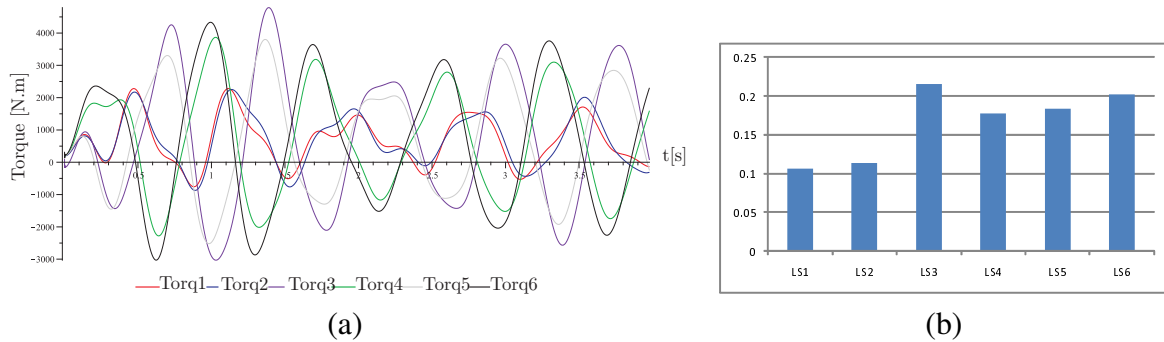


Figure 6: (a) torque distributions on all six yaw drives and (b) load sharing of wind turbine yaw mechanism

7 Conclusions

A dynamics model of multi-motor-gear systems was developed. The model considers of the gear stiffness, shaft stiffness, backlash as well as tooth profile errors. Numerical examples showed that the model is able to reveal the difference of load sharing among paths, hence can be used for further analysis of systems with multi-path transmissions, including the yawing mechanisms. The developed model was applied to a yaw mechanism, for which the load sharing among all yaw drives were simulated. Simulation results show the effectiveness of the model in the analysis of uneven load sharing. Moreover, preliminary parametric study reveals that the load sharing is dependent on the running frequency—a more uneven load sharing is observed with simulations at low frequencies.

Acknowledgement

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